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DYNAMIC MODEL OF AN ANTHROPOMORPHIC ROBOT

BY

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Abstract. The paper presents the dynamic model of the trajectory generating mechanism of an anthropomorphic robot. There was chosen the anthropomorphic robot because the structure of this robot is often used in construction of robots. The recursive formulation Newton-Euler was used to describe the dynamic model.

Key words: dynamic model, anthropomorphic robot, Newton-Euler formulation.

1. Introduction

Generally, the dynamic analysis of robots is a complex problem, requiring a huge amount of calculus. Through direct dynamic analysis of robots is aimed to determine the kinematic parameters of the robot with the hypothesis that the motor moments from driving pairs and the external forces are known.

Most papers in the field have a classic approach for the expression of the motion equations based on Lagrange equations. The first algorithms for inverse dynamic of robots have used Newton-Euler formulation. The most used algorithm is the recursive Newton-Euler algorithm (Featherstone & Orin, 2000). The use of recursive formulation of the Lagrange equation have shown that this

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formulation is slower (requires a bigger time) than the algorithm using recursive formulation Newton-Euler (Sciavicco & Siciliano, 2000).

In this paper, there was realized the dynamic model of the positioning mechanism for an anthropomorphic robot. For dynamic model, the recursive formulation Newton-Euler was used, establishing the motion equation for the positioning mechanism. In the second stage, the motion equation was solved using a numerical method, using the Runge-Kutta algorithm.

Solving the motion equation of the positioning mechanism has allowed establish the variation of the position angles and the angular speeds of the elements forming this mechanism.

2. Recursive formulation Newton-Euler

For the motion analysis and control of a robot, there are known more methods. The most used are: Newton-Euler formulation and Lagrange equation of second species.

With the hypothesis that each element of the robot is a rigid body, then, the distributed mass of each element is completely characterized if there are known the mass center position and the inertia torsor of the element. The required forces for a specified motion are a function depending on the desired acceleration and the mass distribution of elements (Craig, 2005).

In order to apply the Newton-Euler formulation, there is necessary to determine the linear acceleration of the mass center and the angular acceleration of the element.

2.1. Calculus of Speed and Acceleration

In order to calculate the inertia forces and moments acting on robot's elements, there is necessary to calculate the angular speed and acceleration and the linear speed and acceleration of the mass center, for each element (Corke, 2001).

More methods are known to calculate the kinematic parameters of the mass center (McKerrow, 1991). Thus, in order to establish the link between the speed of kinematic pairs and the angular and linear speed of the final effector, the geometric Jacobian matrix is used. In the case of locating the final effector with respect to a reference system using a system of analytical equations, the Jacobian matrix is obtained by derivation of respective equations with respect to the variables of kinematic pairs (Doroftei, 2006).

In this way, the analytical Jacobian matrix is obtained, which differs, in general, by the geometric one. Using these matrixes in calculus of kinematic parameters of mass centers requires a huge amount of calculus.

With the goal of reducing the amount of calculus, there is used the method of “propagation” of speed and acceleration from an element to another one. In this case, the calculus starts with element connected to the robot base.

The kinematic parameters of each element are calculated in a recursive manner starting with the first mobile element and finishing with the final effector.

For the beginning, the angular speed and acceleration are calculated for each element with respect to preceding element.

If we consider two consecutive elements in the structure of a robot, namely i-1 and i, then the angular speed and acceleration are determined with relations:

• *angular speed* (Doroftei, 2006):

− for the rotation pair

$$^{i}\omega_{0,i} = {}^{i}_{i-1}R \cdot ({}^{i-1}\omega_{0,i-1} + {}^{i-1}\omega_{i-1,i}) ; \tag{1}$$

− for the translation pair

$$^{i}\omega_{0,i} = {}^{i}_{i-1}R \cdot {}^{i-1}\omega_{0,i-1} . \tag{2}$$

• *angular acceleration*:

− for the rotation pair

$$^{i}\varepsilon_{0,i} = {}^{i}_{i-1}R \cdot ({}^{i-1}\varepsilon_{0,i-1} + {}^{i-1}\omega_{0,i-1} \times {}^{i-1}\dot{\theta}_{i-1,i} + {}^{i-1}\ddot{\theta}_{i-1,i}) ; \tag{3}$$

− for the translation pair

$$^{i}\varepsilon_{0,i} = {}^{i}_{i-1}R \cdot {}^{i-1}\varepsilon_{0,i-1} . \tag{4}$$

where: $^{i}\omega_{0,i}$ is the angular speed of the reference system {i} with respect to reference system {0}, related to the reference system {i}; $^{i-1}\omega_{0,i-1}$ − the absolute angular speed of the origin O_{i-1} ; $^{i}_{i-1}R$ − rotation matrix for transformation from reference system {i-1} to reference system {i}; $^{i-1}\omega_{i-1,i} = {}^{i-1}\dot{\theta}_{i-1,i}$ − the relative angular speed of kinematic pair {i}, related to reference system {i-1}, with respect to reference system {i-1}; $^{i-1}\ddot{\theta}_{i-1,i}$ − the relative angular acceleration of kinematic pair {i}, related to reference system {i-1}, with respect to reference system {i-1}.

For propagation of linear speeds and accelerations, the following relations are used:

• *linear speeds*

− for rotation pair:

$$ {}^{i}v_{0,i} = {}^{i}_{i-1}R \cdot {}^{i-1}v_{0,i-1} + {}^{i}\omega_{0,i} \times {}^{i}_{i-1}R \cdot {}^{i-1}_{i}p \tag{5} $$

− for rotation pair:

$$ {}^{i}v_{0,i} = {}^{i}_{i-1}R \cdot ({}^{i-1}v_{0,i-1} + {}^{i-1}\omega_{0,i-1} \times {}^{i-1}_{i}p + {}^{i-1}_{i}\dot{p}) \tag{6} $$

where: ${}^{i}v_{0,i}$ is absolute linear speed of origin O_i; ${}^{i-1}v_{0,i-1}$ − absolute linear speed of origin O_{i-1}; ${}^{i-1}_{i}p$ − the translation vector for transformation from reference system {i-1} to reference system {i}; ${}^{i-1}_{i}\dot{p}$ − relative linear speed of kinematic pair {i} related to reference system {i-1}.

• *linear accelerations*

− for rotation pair:

$$ \begin{aligned} {}^{i}a_{0,i} = {}^{i}_{i-1}R \cdot [& {}^{i-1}a_{0,i-1} + {}^{i-1}\varepsilon_{0,i-1} \times {}^{i-1}_{i}p + {}^{i-1}\omega_{0,i-1} \times ({}^{i-1}\omega_{0,i-1} \times {}^{i-1}_{i}p) + \\ & 2 \cdot {}^{i-1}\omega_{0,i-1} \times ({}^{i-1}\dot{\theta}_{i-1,i} \times {}^{i-1}_{i}p) + {}^{i-1}\ddot{\theta}_{i-1,i} \times {}^{i-1}_{i}p + {}^{i-1}\dot{\theta}_{i-1,i} \times ({}^{i-1}\dot{\theta}_{i-1,i} \times {}^{i-1}_{i}p)]; \end{aligned} \tag{7} $$

− for translation pair:

$$ \begin{aligned} {}^{i}a_{0,i} = {}^{i}_{i-1}R \cdot [& {}^{i-1}a_{0,i-1} + {}^{i-1}\varepsilon_{0,i-1} \times {}^{i-1}_{i}p + {}^{i-1}\omega_{0,i-1} \times ({}^{i-1}\omega_{0,i-1} \times {}^{i-1}_{i}p) + \\ & 2 \cdot {}^{i-1}\omega_{0,i-1} \times {}^{i-1}_{i}\dot{p} + {}^{i-1}_{i}\ddot{p}], \end{aligned} \tag{8} $$

where: ${}^{i}a_{0,i}$ is absolute linear acceleration of origin O_i; ${}^{i-1}a_{0,i-1}$ − absolute linear acceleration of origin O_{i-1}; ${}^{i-1}_{i}\dot{p}$ − relative linear speed of kinematic pair {i} related to reference system {i-1}; ${}^{i-1}_{i}\ddot{p}$ − relative linear acceleration of kinematic pair {i} related to reference system {i-1}, with respect to reference system {i-1}.

The linear accelerations of mass centers are determined with relation:

$$ {}^{i}a_{0,Ci} = {}^{i}a_{0,i} + {}^{i}\varepsilon_{0,i} \times {}^{i}p_{i,Ci} + {}^{i}\omega_{0,i} \times ({}^{i}\omega_{0,i} \times {}^{i}p_{i,Ci}), \tag{9} $$

where: ${}^{i}p_{i,Ci}$ is position vector of mass center of element i with respect to origin O_i of reference system {i}.

The gravity acceleration:

$$^{i}g = {}^{i}_{0}R \cdot {}^{0}g\,, \tag{10}$$

2.2. Calculus of Forces and Moments

After the speeds and the acceleration of the kinematic elements were determined, the forces and moments acting on each element can be calculated. The forces and moments acting on element i of a serial robot are represented in Fig. 1.

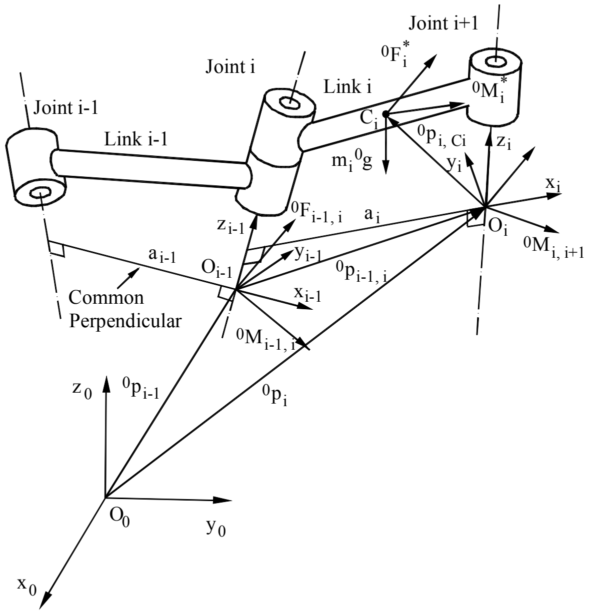

Fig. 1 − The forces and moments acting on the element i.

The calculus of forces and moments starts from the final effector and ends at the fix element. For the beginning, the inertia forces and moments are calculated with the following relations:

$$^{i}F_i^* = -m_i \cdot {}^{i}a_{0,Ci}\,; \tag{11}$$

$$^{i}M_i^* = -{}^{i}I_i \cdot {}^{i}\varepsilon_{0,i} - {}^{i}\omega_{0,i} \times ({}^{i}I_i \cdot {}^{i}\omega_{0,i})\,; \tag{12}$$

where: $^{i}F_i^*$ is the inertia force acting at the mass center of the element i; $^{i}M_i^*$ − the inertia moment acting at the mass center of the element i; $^{i}a_{0,Ci}$ − linear acceleration of the mass center of element i; $^{i}I_i$ − the inertia matrix of element i with respect to its mass center; m_i − the mass of element i.

At the next step, the balance equations for forces and moments acting on element i, are established with respect to the mass center of the element. After the forces and moments were written with respect to reference system {i}, they have to be transformed with respect to reference system {i-1}. This is done with the following relations:

$$ {}^{i}F_{i-1,i} = {}^{i-1}_{i}R \cdot ({}^{i}F_{i,i+1} + m_i \cdot {}^{i}g - {}^{i}F_i^*); \tag{13} $$

$$ {}^{i-1}M_{i-1,i} = {}^{i-1}_{i}R \cdot ({}^{i}M_{i,i+1} + ({}^{i}p_{i-1,i} + {}^{i}p_{i,Ci}) \times {}^{i}F_{i-1,i} - {}^{i}p_{i,Ci} \times {}^{i}F_{i,i+1} - {}^{i}M_i^*); \tag{14} $$

where: ${}^{i}g$ is the vector of gravity acceleration; ${}^{i}p_{i-1,i} = {}^{i}_{i-1}R \cdot {}^{i-1}p_{i-1,i} = {}^{i}_{i-1}R \cdot {}^{i-1}_{i}p$; ${}^{i}p_{i,Ci}$ – the position vector of mass center of element i with respect to origin O_i of the reference system {i}.

The driving moments and the actuator forces are obtained by projecting the constraint forces on the axis of corresponding driving kinematic pair:

– for rotation pair:

$$ \tau_i = {}^{i-1}M_{i-1,i}^T \cdot {}^{i-1}z_{i-1};; \tag{15} $$

– for translation pair:

$$ \tau_i = {}^{i-1}F_{i-1,i}^T \cdot {}^{i-1}z_{i-1}.; \tag{16} $$

3. Numerical Results of Dynamic Model

The numerical results of dynamic model are obtained for an anthropomorphic manipulator represented in Fig. 2.

If we consider the reference systems attached to the kinematic elements as shown in Fig. 2, then the Denavit-Hartenberg parameters for standard convention are those represented in Table 1.

Table 1
The Denavit-Hartenberg Parameters

i	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	L_1	$\theta_1 + \pi/2$
2	L_2	0	0	θ_2
3	L_3	0	0	θ_3

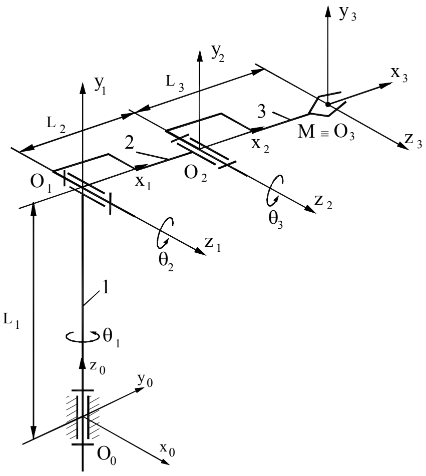

Fig. 2 − Structural scheme of the anthropomorphic manipulator.

Using relations (1) - (4), the angular speeds and accelerations of the kinematic elements of anthropomorphic manipulator are determined. With the relations (7) and (8) the liner accelerations of the mass centers of elements are determined. For symbolic calculus of speeds and accelerations, it is used Toolbox Math Symbolic from MatLab.

For the calculus of inertia forces and moments, the relations (11) and (12) are used, and for the calculus of constraint forces and moments acting in driving pairs the relations (13), (14) are used. These forces and moments lay at the basis of determining the driving moments in driving pairs. The relations for driving moments are used to determine the motion equations of anthropomorphic manipulator.

In the case of anthropomorphic manipulator, all kinematic pairs are rotation pairs and hence, the driving moments from driving pairs are determined with relation (15). The motion equations of the anthropomorphic manipulator are as follows:

$$\begin{aligned}
&[1/2\, L_2^2\, m_3 + 0.083 L_1^2 \cdot m_1 + 1/2\, L_2\, L_3\, m_3 \cos(\theta_3) + 1/6\, L_3^2\, m_3 + \\
&+1/6 L_2^2 \cdot m_2 + 1/2\, L_2^2\, m_3 \cos(2\cdot\theta_2) + 1/6 L_3^2\, m_3 \cos(2\cdot\theta_2 + 2\cdot\theta_3) + \\
&+1/6 L_2^2\, m_2 \cos(2\cdot\theta_2) + 1/2\, L_2\, L_3\, m_3 \cos(2\theta_2 + \theta_3)] \cdot \ddot{\theta}_1 + \\
&+[-1/3 L_3^2\, m_3 \sin(2\cdot\theta_2 + 2\cdot\theta_3)\cdot\dot{\theta}_1\cdot\dot{\theta}_2 - \\
&-1/2 L_2\, L_3\, m_3 \sin(2\theta_2 + \theta_3)\cdot\dot{\theta}_1\cdot\dot{\theta}_3 - L_2\, L_3\, m_3 \sin(2\theta_2 + \theta_3)\cdot\dot{\theta}_1\cdot\dot{\theta}_2 - \\
&-L_2^2\, m_3 \sin(2\cdot\theta_2)\cdot\dot{\theta}_1\cdot\dot{\theta}_2 - 1/3 L_2^2\, m_2 \sin(2\cdot\theta_2)\cdot\dot{\theta}_1\cdot\dot{\theta}_2 - \\
&-1/3 L_3^2\, m_3 \sin(2\cdot\theta_2 + 2\cdot\theta_3)\cdot\dot{\theta}_1\cdot\dot{\theta}_3 - 1/2\, L_2\, L_3\, m_3 \sin(2\theta_3)\cdot\dot{\theta}_1\cdot\dot{\theta}_3] + \\
&+[-L_3\, F_{3_4z} \cos(\theta_2 + \theta_3) - L_2\, F_{3_4z} \cos(\theta_2)] = \tau_1 \; .
\end{aligned} \tag{17a}$$

$$\begin{aligned}
&[\, L_2 L_3 m_3 \cos(\theta_3) + L_2^2 m_3 + 1/3 L_2^2 \cdot m_2 + 1/3\, L_3^2 m_3 \,] \cdot \ddot{\theta}_2 + \\
&+[1/2\, L_2 L_3 m_3 \cos(\theta_3) + 1/3 L_3^2 \cdot m_3] \cdot \ddot{\theta}_3 + [1/2 L_2^2 m_3 \sin(2\cdot\theta_2)\cdot\dot{\theta}_1^2 + \\
&+1/2 L_2 L_3 m_3 \sin(2\theta_2+\theta_3)\cdot\dot{\theta}_1^2 - 1/2 L_2 L_3 m_3 \sin(\theta_3)\cdot\dot{\theta}_3^2 - \\
&-L_2 L_3 m_3 \sin(\theta_3)\cdot\dot{\theta}_2\cdot\dot{\theta}_3 + 1/6 L_2^2 m_2 \sin(2\cdot\theta_2) + \\
&+1/6 L_3^2 m_3 \sin(2\cdot\theta_2 + 2\cdot\theta_3)\cdot\dot{\theta}_1^2] + [L_2 m_3 g \cos(\theta_3) + L_2 F_{3_4y} \cos(\theta_3) \\
&+L_2 F_{3_4x} \sin(\theta_3) + 1/2 L_2 m_2 g + 1/2 L_3 m_3 g + L_3 F_{3_4y}] = \tau_2 \;.
\end{aligned} \tag{17b}$$

$$\begin{aligned}
&[\, 1/3 L_3^2 m_3 + 1/2 L_2 L_3 m_3 \cos(\theta_3)] \cdot \ddot{\theta}_2 + 1/3\; L_3^2 \cdot m_3 \cdot \ddot{\theta}_3 + \\
&+[1/4 L_2 L_3 m_3 \sin(2\cdot\theta_2+\theta_3)\cdot\dot{\theta}_1^2 + 1/6 L_3^2 \cdot m_3 \sin(2\theta_2 + 2\theta_3)\cdot\dot{\theta}_1^2 - \\
&-1/4 L_2 L_3 m_3 \sin(\theta_3)\cdot\dot{\theta}_1^2 + 1/2 L_2 L_3 m_3 \sin(\theta_3)\cdot\dot{\theta}_2^2] + \\
&+[L_3 F_{3_4y} + 1/2 L_3 m_3 g] = \tau_3 \;.
\end{aligned} \tag{17c}$$

The motion equations for the anthropomorphic robot can be written also under matrix form:

$$M(\theta_1,\theta_2,\theta_3)\cdot\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\\ \ddot{\theta}_3\end{bmatrix} + M(\theta_1,\theta_2,\theta_3,\dot{\theta}_1,\dot{\theta}_2,\dot{\theta}_3) + G(\theta_1,\theta_2,\theta_3) = \begin{bmatrix}\tau_1\\ \tau_2\\ \tau_3\end{bmatrix}. \tag{18}$$

With the hypothesis that the matrix of masses M is nonsingular (nonzero determinant), then relation (18) can be written as:

$$\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\\ \ddot{\theta}_3\end{bmatrix} = M^{-1}\left(\begin{bmatrix}\tau_1\\ \tau_2\\ \tau_3\end{bmatrix} - \mathrm{N} - \mathrm{G}\right). \tag{19}$$

For dynamic modelling of the displacement of the anthropomorphic robot, a calculus code was realized. That calculus code was run under MatLab. The motion equations were solved through the numerical method Runge-Kutta of fourth order.

After solving the motion equations, there are determined the variation of position angles of the elements and the variation of angular speeds of the elements.

For a given time interval, in Fig. 3 there is presented the variation of position angles of the elements. Also, in Fig. 4 there is presented the variation of angular speeds of the elements.

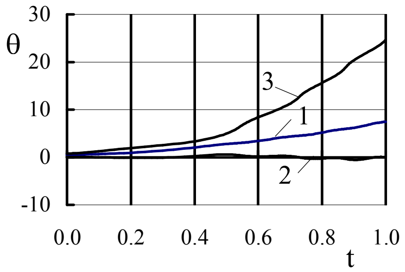

Fig. 3 − Variation of position angles.

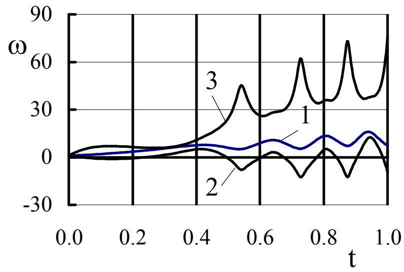

Fig. 4 − Variation of angular speed.

For dynamic modelling of the displacement of the anthropomorphic robot, a calculus code was realized. That calculus code was run under MatLab. The motion equations were solved through the numerical method Runge-Kutta of fourth order.

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For a given time interval, in Fig. 3 there is presented the variation of position angles of the elements. Also, in Fig. 4 there is presented the variation of angular speeds of the elements.

4. Conclusions

In general, for a robot, the direct dynamic analysis implies a huge amount of calculus. In order to decrease the amount of calculus, the recursive formulation Newton-Euler was used.

By using the proposed dynamic model, there are obtained information regarding if the moments of actuators are big enough to put the robot to work. Also, there is established if the external load applied to the final element does not block the work of the robot.

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MODELUL DINAMIC AL UNUI ROBOT ANTROPOMORF

(Rezumat)

În lucrare se prezintă modelul dinamic al mecanismului generator de traiectorie pentru un robot antropomorf. A fost ales robotul antropomorf, deoarece structura acestui robot este des folosită în construcția roboților. Pentru realizarea modelului dinamic s-a folosit formularea recursivă Newton Euler. În prima etapă au fost determinați parametrii cinematici ai centrelor de masă şi apoi au fost determinate ecuațiile de mişcare ale elementelor robotului. În etapa a doua au fost rezolvate aceste ecuații folosind algoritmul de calcul numeric Runge Kutta.